

Homework 4

In problem 1, we look at how the inversion layer developed at night goes away in the morning by solar heating. Problem 2 is a follow up of the first problem, in which we consider the effect of latent heat release. Problems 3 and 4 are exercises of first-order and non-local closure schemes.

1. Destruction of the inversion layer (10 points)

Consider a night time inversion layer at 0-500 m. The surface temperature is 290 K. Temperature increases with altitude at 3 K/km.

(a) As the sun rises, heat absorption at the surface destroys the inversion layer. At time t_0 , the whole inversion is gone. What is the temperature at the surface at t_0 ? Show the vertical profile of T from the surface to 1-km altitude at t_0 .

(b) Assume that the surface is dry, the sensible heat flux at 500 m is zero, the heat flux into the ground is zero, and there is no overhead cloud. Assuming that it takes 3 hours after sunrise to destroy the inversion layer, calculate the amount of solar radiation absorbed per unit area during the process. (Hint: use the empirical formula for IR energy loss discussed in class.)

2. Effects of soil moisture (10 points)

We now consider the effects of soil moisture. Assume that the relative humidity is constant at 0.6. The surface albedo is 0.1.

(a) Estimate the latent heat release per unit area from sunrise to when the inversion layer is gone (t_0'). For the sake of simplifying the computation, use the average temperature of the layer to compute moisture contents at sunrise and t_0' . (Hint: Saturation vapor pressure can be estimated with $e_s = 6.112 \exp\left(\frac{17.67T}{T + 243.5}\right)$, where e_s is in mb and T is in °C.)

(b) If the average solar elevation during this period is 40°, estimate how much longer the temperature inversion at 500 m persists over moist than dry soils ($t_0' - t_0$). (Hint: (1) use the empirical formulas discussed in class; (2) while sensible and latent heat fluxes occur at the same time, calculations of their effects can be done separately.)

(c) Estimate the average Bowen ratio at the surface during the destruction of the inversion layer.

3. 1st-order closure (5 points)

Given

Z	12	8	2	0.1= z_0	(m)
$\bar{\theta}$	300	301	303	308	(K)
\bar{U}	5.4	5.0	3.4	0	(m/s)

Use 1st-order closure to find $\overline{w'\theta'}$ and $\overline{u'w'}$ at $z=z_0$ and at $z=10$ m. (Hint: Use

$$K_m = \kappa^2 z^2 \left[(\partial \bar{U} / \partial z) + \left(g / \bar{\theta} \left| \partial \bar{\theta} / \partial z \right| \right)^{1/2} \right]$$

4. Non-local closure (10 points)

Given the following transilient matrix,

$$c_{ij}(\Delta t = 15 \text{ min}) = \begin{bmatrix} 0.7 & 0 & 0.2 & 0.1 \\ 0.1 & 0.6 & ? & 0.2 \\ ? & ? & ? & ? \\ 0.1 & 0.2 & ? & 0.3 \end{bmatrix}$$

- a) Fill the missing elements to yield an allowable matrix.
 b) Given an initial tracer distribution of 100 ppbv at the lowest grid box ($i=1$), 10 ppbv in the top grid box ($i=4$), and no tracer else where, find the tracer distribution after 15 minutes. (Assume that the volumes of the 4 grid boxes are the same.)

Hint:

$$\begin{bmatrix} x_1^{new} \\ x_2^{new} \\ x_3^{new} \\ x_4^{new} \end{bmatrix} = \begin{bmatrix} 0.7 & 0 & 0.2 & 0.1 \\ 0.1 & 0.6 & ? & 0.2 \\ ? & ? & ? & ? \\ 0.1 & 0.2 & ? & 0.3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$